Sampled-data energetic management of a fuel cell/supercapacitor system

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***Abstract*— A non linear passivity-based control can be suitably used to achieve stability of a Parallel Hybrid Fuel Cell/Supercapacitor (FC/SC) Power Source Architecture. Nevertheless, passivity properties are usually lost when such a controller is implemented in a sampled-data context and, as a consequence, stabilization objectives are degraded. This paper proposes a direct sampled-data control strategy based on the IDA-PBC techniques, ensuring the stabilization of the (FC/SC) system at its desired equilibrium point. Such a controller guarantees good energetic performances for a large range of sampling periods and small current amplitudes. Arguing so such a digital controller avoids the premature usage of FC and SCs, maintains a good transient response and can be implemented by cheaper computers, dealing important industrial requirements.**

I. INTRODUCTION

In the last years, in function of increasing social preoccupations and rigorousness of environmental norms, the electrical vehicle emerges as the main challenge of the international automobile industry. However, the principal difficulty, which prevents its rapid dissemination, concerns the development of an efficient and cheap energy storage system, able to provide a large range of power and to ensure a great vehicle's autonomy. An interesting solution consists in a hybrid power source constructed by the interconnection of a hydrogen fuel cell (FC) based on proton exchange membrane and supercapacitors (SCs) [4], [12], [13], [5]. The FC constitutes an attractive primary energy source, providing a suitable autonomy with no local pollution. In parallel, SCs are able to provide a short pulse of energy for the fast car power demands, which avoids the usage of the FC in high powers, increasing its live time, and permits the implementation of a regenerative braking energy recovery system.

Several electronic structures have been proposed to connect FC and SCs through a DC bus into the electrical load representing vehicle power demands, with a clear advantage to the parallel architecture (easier power flow management, higher reliability and the lower stress level of the system components). To ensure a good DC voltage regulation, single or double-converters structures can be used [1], [2]. The second strategy allows better control but increases energetic losses while the first one ensures less energetic losses but with a worst DC voltage regulation.

In this paper our study is limited to the double-converter structure. In such architecture FC and SCs are connected to the DC bus through two independent DC/DC converters. The FC converter permits a unilateral power flow achieving two objectives: to supply the load power demands and to charge the SC. The SCs converter allows a bi-lateral power flow permitting in addition to supply the load, the SCs recharge, which can be accomplished directly by the FC or by the vehicle regenerative braking system.

The FC/SC energetic management is realized by a controller which must ensure three major objectives: to respect the FC dynamic (limiting the sudden variations in the current); the control of the charge of storage devices (SCs); and the power response (positive or negative) required to the load.

Several linear and non linear techniques can be used to pilot each converter, controlling the DC bus voltage/SCs charge and ensuring that the FC delivers only a slowly varying current (the power transient must be guaranteed by the SCs). In [3] and [6] it was proposed the well known Interconnection and Damping Assignment - Passivity Based Control (IDA-PBC) (see [11] for a theoretical study) for the FC/SC system energetic management. Such a methodology provides, beyond a powerful controller ensuring the fixed objectives, a formal proof of global system stability, which is mandatory especially in embedded and transportation applications.

Nevertheless, in a practical way, when the controller is implemented by a computer, the system is placed in a sampled-data context and, as it is well known, passivity properties are usually lost [9]. Consequently, passivity based controllers implemented through a zero order holder device (emulation process) loose their validity. This problem is often masked by the choice of a very small sampling period and/or very small controller gain value. However, this simplistic solution immediately reflects in unrealistic industrial requirements such as the necessity of expansive computers with remarkable capacity, slower system transient response (resulting in a less performing vehicle) and mainly the need of increasing current amplitudes.

In this study a direct sampled-data IDA-PBC design to control the fuel cell/supercapacitor system is proposed aiming to achieve global stabilization with lower current demands being valid in a large range of sampled periods. Motivated by a recent theoretical result in [14], a digital passivity-based controller is used to the FC/SCs energy management. An analysis concerning sampling effects on the control requirements is also performed.

The paper is organized as follows. Section II introduces the chosen electronic architecture and gives its model by considering converter average dynamics. The sampled-data IDA-PBC strategy is presented in section III. The benefits of the proposed controller are illustrated by simulations in section IV.

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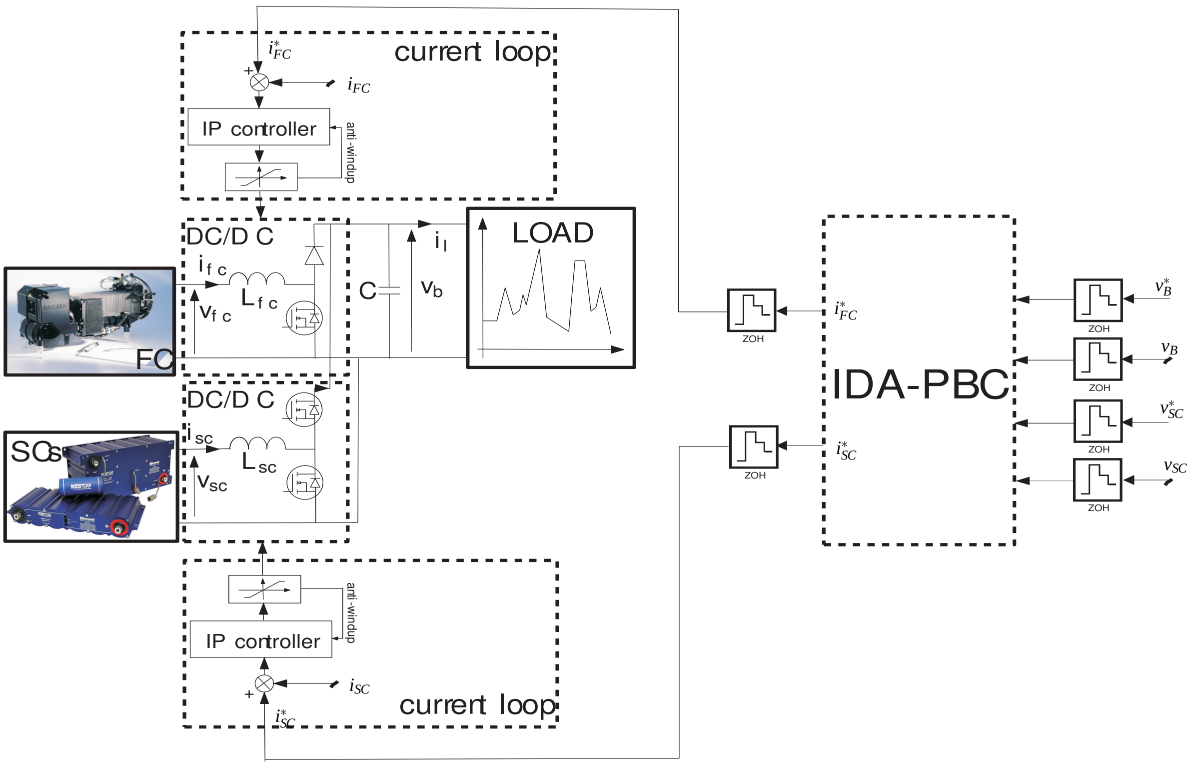

Fig. 1. Sampled-data IDA-PBC structure

II. System Architecture

A. Problem settlement

The used electronic structure consists in a cascade scheme proposed in [2] and represented in Fig. 1. Any vehicle's power requirement (speed variations, mass elevation, tire friction and aerodynamic dissipations) results in an electrical power demand. Such an architecture permits a *frequency decoupling* between the sources to supply this power [2]:

- High frequencies, i.e. above the *kHz* are filtered by the DC bus capacitor C;
- Medium frequencies (from *kHz* to *Hz*) are ensured by the SCs associated to a *Boost Converter*. The choice of such a reversible power converter permits the dynamical charge and recharge of the SCs;
- Low frequency (less than 1*Hz*) are ensured by the FC associated to a *Boost Converter*, ensuring a slowly varying current delivered by the FC which guarantees its durability.

A control system must be designed to maintain the DC bus voltage with a constant value equal to its reference v_B^*, ensuring that currents i_{FC} and i_{SC} stay in a defined range (in order to protect the sources, converters and the load) for any load variation. Such a control scheme is devised in three sub-loops:

- A digital non linear IDA-PBC loop, controlling the DC bus voltage and the charge of SCs by fixing piecewise constant current references i_{FC}^* and i_{SC}^* values;
- Two fast inner IP current loop with an anti-windup, ensuring that from any load requirements, i_{FC} and i_{SC} go rapidly to their reference values i_{FC}^* and i_{SC}^*, fixed by the digital IDA-PBC. The anti-windup loop ensures that their values do not exceed i_{FCmax} and i_{SCmax}, defined in order to protect FC and SCs.

B. System Model

Since i_{FC} and i_{SC} are controlled by two fast analogical inner loops, their dynamics can be neglected and the current values will be assumed equal to the their reference given by digital IDA-PBC in this paper. At the FC level, as the chemical reaction is faster than electrical dynamics, the FC static model is used and its voltage v_{FC} is computed according to the i_{FC} value from a 5$^{\text{th}}$ order polynomial function as in [6]. The power converters, the DC bus and the load are modeled in the sequel.

1) FC-Boost Converter: Since the FC voltage v_{FC} is often lower than the DC bus reference voltage v_B^*, the converter must increase v_FC aiming to maintain the DC bus voltage constant equal to its reference for any i_{FC}. Such a converter is controlled by a binary command and α_1 defines its duty cycle. Considering that switches are ideal and a piecewise constant current $i_{FC} = i_{FC}^*$ is ensured by its fast inner current loop, the FC-converter output voltage v_B is given by $v_B = \frac{1}{1-\alpha_1} v_{FC}$ and its dynamics can be, at each sampling interval, represented by the power converter average model:

$$\frac{\mathrm{d}v_B}{\mathrm{d}t} = \frac{1}{C}\left(\frac{v_{FC}}{v_B} i_{FC} - i_L\right), \tag{1}$$

with i_L the DC current delivered to the load.

2) SCs-Boost Converter: Such a converter permits two different operations: the buck operating mode when the SCs receive energy from DC bus, and a boost operating mode when the SCs provide energy to the DC bus. Converter transistors are piloted by an inverse binary command and α_2 defines its duty cycle. Considering that switches are ideal and a piecewise constant current $i_{SC} = i^*_{SC}$ is ensured by its fast inner current loop, the output voltage v_B is given by $v_B = \frac{1}{1-\alpha_2} v_{SC}$ and its dynamics can be represented, during each sampled interval, by the power converter average model:

$$\frac{dv_{SC}}{dt} = -\frac{i_{SC}}{C_{SC}}. \tag{2}$$

3) Load: The load is represented by a $R_L(t)L_LE_L(t)$ series circuit, whose resistance $R_L(t)$ varies depending on the vehicle power demands, $E_L(t)$ represents an f.e.m. of an electrical machine (during the vehicle regenerative breaking process, $E_L(t) > 0$ and the electric load operates as a generator).

C. Complete model

Considering a constant bus capacitance C, $u_1 = i_{FC}$ and $u_2 = i_{SC}$, the complete 3^{rd} order state space model is written, from (1) and (2), as:

$$\dot{v}_B = \frac{1}{C}\left(\frac{v_{FC}}{v_B}u_1 + \frac{v_{SC}}{v_B}u_2 - i_L\right) \tag{3}$$

$$\dot{v}_{SC} = \frac{-u_2}{C_{SC}} \tag{4}$$

$$\dot{i}_L = \frac{-R_L i_L - E_L + v_B}{L_L} \tag{5}$$

The desired equilibrium point is given by $\bar{x} = [\bar{v_B}\ \bar{v}_{SC}\ \bar{i_L}] = \left[v^*_B\ v^*_{SC}\ \frac{v^*_B - E_l(t)}{R_L(t)}\right]$. v^*_B and v^*_{SC} are respectively the DC bus and the desired SCs voltages. Note that i_L has a dynamical equilibrium point, depending of the vehicle power requirements.

Even if the free evolution (i.e. uncontrolled dynamics - $u_1 = u_2 = 0$) of the system (3)-(5) is completely linear, non linearities appear in its forced dynamics. Therefore to design a controller ensuring global asymptotic stabilization at $\bar{x}$, non linear techniques will be employed. Since the measures of the states are obtained under sampling and the control inputs u_1 and u_2 are implemented through a zero order holder device, a direct sampled-data design has to be set to preserve a high performance level. Motivated by the continuous time IDA-PBC design, recently proposed in [6], and for the last theoretical sampled-data result, presented in [14], the chosen methodology consists in a *direct sampled-data IDA-PBC design*.

III. SAMPLED-DATA IDA-PBC

In a continuous-time context, the Passivity Based Control-PBC methodology, introduced in [10], has been further developed versus the Interconnection and Damping Assignment - Passivity Based Control -IDA-PBC ([11] and the references therein) so renewing stabilizing strategies. The basic philosophy is to shape a desired internal structure and then to inject damping to dissipate the total system's energy.

Nevertheless, since passivity is usually lost under sampling, a direct sampled-data study, permitting the design of digital IDA-PBC in a quite systematic way, has been performed in [14]. The sampled-data design is worked out in such a way to match, at the sampling instants, some key behaviors of the continuous-time design. Arguing so, assuming the existence of a continuous-time IDA-PBC controller one looks for a sampled-data controller matching the energectic behavior of the closed loop system.

A. The continuous-time IDA-PBC strategy

Consider a Port Controlled Hamiltonian-PCH dynamics

$$\dot{x}(t) = f(x) + g(x)u = [\mathscr{J}(x) - \mathscr{R}(x)]\nabla H + g(x)u \tag{6}$$

where $x \in R^n$ is the state vector, $u \in R^m$ the control vector with $m < n$, $H : R^n \to R$ is the total energy, $\mathscr{J}(x) = -\mathscr{J}^T(x), \mathscr{R}(x) = \mathscr{R}^T(x) \geq 0$ are the usual interconnection and damping matrices respectively. Let us recall the IDA-PBC design procedure proposed in [11] for such a system's class.

Proposition 3.1: Let (6) and assume the existence of matrices $g^T(x), \mathscr{J}_d(x) = -\mathscr{J}_d^T(x), \mathscr{R}_d(x) = \mathscr{R}_d^T(x) \geq 0$ and a function $\mathscr{H}_d : R^n \to R$ that verify the Partial Derivative Equation-PDE

$$g^{\perp}(x)f(x) = g^{\perp}(x)[\mathscr{J}_d(x) - \mathscr{R}_d(x)]\nabla H_d, \tag{7}$$

where $H_d(x)$ is the target energy, such that $\bar{x} = argmin H_d(x)$ with $\bar{x}$ the equilibrium to be stabilized and $g^{\perp}(x)$ is a left annihilator of $g(x)$, i.e. $g^{\perp}(x)g(x) = 0$. Setting

$$u = g^{+}(x)\left((\mathscr{J}_d(x) - \mathscr{R}_d(x))\nabla H_d - f(x)\right), \tag{8}$$

where g^+ is the Moore-Penrose inverse of $g(x)$, i.e. $gg^+ := g[g^T(x)g(x)]^{-1}g^T(x) = I_d$, the closed-loop dynamics has a locally stable equilibrium in $\bar{x}$. If in addition $\bar{x}$ is an isolated minimum of $\mathscr{H}_d(x)$ and if the largest invariant set under the closed loop dynamics contained in

$$\{x \in R^n s.t. [\nabla \mathscr{H}_d]^T \mathscr{R}_d(x) \nabla \mathscr{H}_d = 0\}$$

equals $\{\bar{x}\}$, then $\bar{x}$ is asymptotically stable. An estimate of its domain of attraction is given by the largest bounded level set $\{x \in R^n s.t. \mathscr{H}_d(x) \leq c\}$.

B. Continuous-time objectives

A detailed continuous-time design is presented in [6].

Rewriting the system (3)-(5) in the PCH form as

$$\dot{x} = (\mathscr{J} - \mathscr{R})\nabla\mathscr{H} + g_1(x)u_1 + g_2(x)u_2 + D$$

with the interconnection and the dissipation matrices given respectively by

$$\mathscr{J} = \begin{bmatrix} 0 & 0 & -\frac{1}{L_L C} \\ 0 & 0 & 0 \\ \frac{1}{L_L C} & 0 & 0 \end{bmatrix} \text{ and } \mathscr{R} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \frac{R_L}{L_L^2} \end{bmatrix};$$

the system storage function is

$$\mathscr{H} = \frac{1}{2}x^T Q x, \quad \text{with } Q = \begin{bmatrix} C & 0 & 0 \\ 0 & C_{SC} & 0 \\ 0 & 0 & L_L \end{bmatrix};$$

the control matrices g_1 and g_2 are defined as

$$g_1 = \begin{bmatrix} \frac{1}{C}\frac{v_{FC}}{v_B} \\ 0 \\ 0 \end{bmatrix}, \quad g_2 = \begin{bmatrix} \frac{1}{C}\frac{v_{SC}}{v_B} \\ -\frac{1}{Csc} \\ 0 \end{bmatrix},$$

and the f.e.m E_L is interpreted as a perturbation defined by the vector $D^T = [0\ 0\ -E_L/L_L]$.

Considering the continuous time control laws proposed in [6]:

$$u_{c1} = \frac{v_B}{v_{FC}}\left(\frac{\bar{v}_B}{R_L} + \left(K_1\frac{v_{SC}}{v_B} - K_2\right)\tilde{v}_B - K_1\tilde{v}_{SC}\right) \quad (9)$$

$$u_{c2} = -K_1\tilde{v}_B, \quad (10)$$

with strictly positive gains K_1 and K_2 and $[\tilde{v}_B, \tilde{v}_{SC}, \tilde{i_L}] = [v_B - \bar{v}_B, v_{SC} - \bar{v}_{SC}, i_L - \bar{i}_L]$. The closed-loop system can be written:

$$\dot{x_c} = [\mathscr{J}_d - \mathscr{R}_d]\nabla\mathscr{H}_d(x_c) + D_d \quad (11)$$

where $\mathscr{H}_d(x_c) = (x_c - \bar{x})^T Q (x_c - \bar{x})$ is the desired storage function having a minimum at the equilibrium point $\bar{x}$. The index $_c$ on x_c indicates that the system is under the continuous-time control law $u_c^T = [u_{c1}\ u_{c2}]^T$. The new interconnection and dissipation matrices are written as follows:

$$\mathscr{J}_d = \begin{bmatrix} 0 & -\frac{K_1}{CC_{SC}} & -\frac{1}{L_L C} \\ \frac{K_1}{CC_{SC}} & 0 & 0 \\ \frac{1}{L_L C} & 0 & 0 \end{bmatrix} \quad \text{and} \quad \mathscr{R}_d = \begin{bmatrix} \frac{K_2}{C^2} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \frac{R_L}{L_L^2} \end{bmatrix}$$

and the perturbation matrix $D_d^T = [E_L/(R_l C)\ 0\ 0]$.

With $E_L = 0$, the time-derivative of $\mathscr{H}_d$ is given by:

$$\dot{\mathscr{H}_d}(x_c) = -(\nabla\mathscr{H}_d(x_c))^T \mathscr{R}_d \nabla\mathscr{H}_d(x_c), \quad \forall x_c \neq \bar{x}$$

and as $\mathscr{H}_d(x_c)$ has a global minimum in $\bar{x}$, by invariance principle of LaSalle theorem, $\bar{x}$ is a global asymptotic stable equilibrium point of the closed-loop system (11) [7]. Considering that $E_L \neq 0$ only during finite time intervals T_b, $T_b = [t_1, t_2]$, s.t. $t_1 < t_2 < \infty$, the term D_d appears as an additive term

$$\frac{\tilde{v}_B E_l}{R_l}$$

disturbing $\dot{\mathscr{H}_d}(x_c)$. With this extra power the system leaves momentarily the equilibrium $\bar{x}$. However the damping assignment character of the controller u_c makes SCs to absorb this additive power, "re-stabilizing" asymptotically the closed-loop system after t_2.

C. Sampled-data conception

Analyzing the interconnection and dissipation matrices one notes that u_{c1} and u_{c2} improve damping of bus voltage through the term $\frac{K_2}{C^2}$. Then, by changing the system interconnection structure, this damping assignment is propagated to v_{SC} ensuring the stabilization at the desired equilibrium point $\bar{x}$ with a good transient response. This energetic behavior must be preserved under sampling by the digital controller.

The closed-loop continuous-time state is denoted by x_c, as in (11). The sampled-time state under a piecewise controller u_k is denoted, at the instant $t = k\delta$ by x_k , its value at the instant $t = (k+1)\delta$ is denoted by x_{k+1}. In this sense, if there exists a u_k matching the energetic behavior of u_c, the following equation must be verified for all x_k by setting $x_c(t = k\delta) = x_k$:

$$\mathscr{H}_d(x_{k+1}) - \mathscr{H}_d(x_k) = \int_{k\delta}^{(k+1)\delta} \dot{\mathscr{H}_d}(x_c(\tau))\mathrm{d}\tau. \quad (12)$$

The left hand side of (12) concerns the desired energetic evolution of the sampled-data system and can be computed as follows:

$$\mathscr{H}_d(x_{k+1}) - \mathscr{H}_d(x_k) = \frac{1}{2}\left(e^{\delta((\mathscr{J}-\mathscr{R})Qx+g(.)u_k)} - Id\right)(x_k - \bar{x})^T Q (x_k - \bar{x}) \quad (13)$$

where $e^f(.) := 1 + \sum_{i\geq 1}\frac{\mathrm{L}_f^i}{i!}$, is the Lie series operator associated with a given vector field $f(.)$, Id the identity operator and $\mathrm{L}_f(.) = \sum_{i=1}^n f_i(.)\frac{\partial}{\partial x_i}$, the usual Lie derivative operator associated with a given vector field $f(.)$ on R^n (see [8] for more details). The right hand size of (12) concerns the energetic evolution of x_c and can be exactly computed as:

$$\begin{aligned} \int_{k\delta}^{(k+1)\delta} \dot{\mathscr{H}_d}(x_c(\tau))\mathrm{d}\tau &= \mathscr{H}_d(x_c(t = (k+1)\delta)) - \mathscr{H}_d(x_c(t = k\delta)) \\ &= x_k^T\left(\left(e^{\delta(\mathscr{J}_d - \mathscr{R}_d)Q}\right)^T Q e^{\delta(J_d - R_d)Q} - Q\right)x_k. \end{aligned}$$

The sampled-data controller u_k is described by its series expansion in δ around the continuous-time one $u_{d0} = u_c|_{t=k\delta}$ as $u_k^\delta = u_{d0} + \sum_{i\geq 1}\frac{\delta}{(i+1)!}u_{di}$, and each so-called "corrective" term u_{di} is computed by comparing homogeneous terms in powers of δ in the equation (12). The calculus of an exact solution being impossible, due to nonlinearities on the original continuous-time system, an interesting solution can proposed at the first approximated order, i.e. $u_k^{\delta T} = [u_{k1}^\delta\ u_{k2}^\delta]^T$ with

$$u_{k1}^\delta = \frac{v_B}{v_{FC}}\left(\frac{\bar{v}_B}{R_L} + \left(K_1\frac{v_{SC}}{v_B} - K_2\right)\tilde{v}_B - K_1\tilde{v}_{SC}\right)\Bigg|_{t=k\delta} \quad (14)$$

$$+\frac{\delta}{2!}\gamma(v_B, v_{SC}, i_L, R_l)\Bigg|_{t=k\delta} \quad (15)$$

$$u_{k2}^\delta = \left(-K_1\tilde{v}_B + \frac{\delta}{2}\frac{K_1}{C}\left(K_2\tilde{v}_B + K_1\tilde{v}_{SC} + \tilde{i}_L\right)\right)\Bigg|_{t=k\delta} \quad (16)$$

and $\gamma(v_B, v_{SC}, i_L)$ given by:

$$\begin{aligned} \gamma &= \dot{u_{c1}} = -K_1^2\frac{\bar{v}_B\tilde{v}_B}{C_{SC}v_{FC}} \\ &\quad -\frac{1}{C_{SC}v_{FC}}\left(K_2\tilde{v}_B + K_1\tilde{v}_{SC} + \tilde{i}_L\right)\left((2v_B - \bar{v}_B)\left(\frac{1}{R_L} - K_2\right) + K_1\bar{v}_{SC}\right) \end{aligned}$$

Proposition 3.2: Given the closed-loop continuous-time system (11), there exists a digital control law $u_k^{\delta T} = [u_{k1}^\delta\ u_{k2}^\delta]^T$ computed as in (15)-(16) which reproduces the continuous-time energetic behavior in δ^2 (error in $O(\delta^3)$), and thus asymptotic stability of $\bar{x}$.

Proof: The sketch of the proof is worked out by replacing $u_k = u_k^\delta$ in (12). It is immediately verified that considering u_k^δ computed as in (15)-(16), the equality (12) is ensured into δ^2. ■

Remark 3.1: The digital controller u_k^δ also maintains the continuous-time interconnected structure (11) under sampling into δ^2 [14] and in this sense damping assignment and propagation through system's states are ensured.

IV. Practical Issues

Since the fuel cell current reference is fixed by u_{k1}^δ and its terms depend of $1/R_l$, for each R_l variation, the sampled-data IDA-PBC controller ensures the DC voltage regulation by varying i_{FC}. Logically, it consists in a undesirable control behavior [6] and to avoid it, the sensibility of i_{FC} in R_l can be regulated by an extended controller computed as:

$$R^{\delta}_{k+1} = e^{-\delta K_{Rl}} R^{\delta}_{k} + (1 - e^{-\delta K_{Rl}}) \frac{i_{Lk}}{v_{Lk}} \tag{17}$$

$$u^{\delta}_{k1} = \frac{v_B}{v_{FC}} \left(\frac{\bar{v}_B}{R^{\delta}_k} + \left(K_1 \frac{v_{SC}}{v_B} - K_2 \right) \tilde{v}_B - K_1 \tilde{v}_{SC} \right) \Bigg|_{t=k\delta} \tag{18}$$

$$+ \frac{\delta}{2!} \gamma(v_B, v_{SC}, i_L, R^{\delta}_k) \Big|_{t=k\delta}. \tag{19}$$

The gain K_{Rl} regulates the sensibility of u_{k1} in R_l, and by choosing a small value for this parameter, a slow i_{FC} variation is guaranteed even for strong changes in R_l. In this case, the DC voltage regulation is ensured by the supercapacitors through variations of the current i_{SC}. Since the variation of i_{FC} are smooth, the corrective term $\gamma(v_B, v_{SC}, i_L, R^{\delta}) = \dot{u}_{c1}$ assumes small values and can be neglected in a practical implementation; at the same time the corrective term of u^{δ}_{k2}, which depends of $\dot{u}_{c2}$, assume high values and becomes very important for the DC bus voltage regulation under sampling.

In this way the practical proposed sampled-data controller $u^{\delta}_{kp} = \left[u^{\delta}_{kp1} \; u^{\delta}_{kp2} \right]$ is computed in terms of:

$$R^{\delta}_{k+1} = e^{-\delta K_{Rl}} R^{\delta}_{k} + (1 - e^{-\delta K_{Rl}}) \frac{i_{Lk}}{v_{Lk}} \tag{20}$$

$$u^{\delta}_{kp1} = \frac{v_B}{v_{FC}} \left(\frac{\bar{v}_B}{R^{\delta}_k} + \left(K_1 \frac{v_{SC}}{v_B} - K_2 \right) \tilde{v}_B - K_1 \tilde{v}_{SC} \right) \Bigg|_{t=k\delta} \tag{21}$$

$$u^{\delta}_{kp2} = \left(-K_1 \tilde{v}_B + \frac{\delta}{2} \frac{K_1}{C} \left(K_2 \tilde{v}_B + K_1 \tilde{v}_{SC} + \tilde{i}_L \right) \right) \Bigg|_{t=k\delta} \tag{22}$$

V. Simulation and Result Analysis

Simulations are performed considering that inner current loops are sufficiently fast and currents i_{FC} and i_{SC} have no limitation in amplitude. The objective is to compare the behavior of the systems under the classical implementation of the continuous-time controller (9)-(10) through a zero-order holder, i.e. *emulated controller* and the proposed extended sampled-data controller (20)-(22). In both cases the sensibility of the fuel cell current reference must be regulated by the gain K_{Rl}, and thus the emulated controller $u_{d0} = [u_{d01} \; u_{d02}]$ is computed in terms of:

$$R^{\delta}_{k+1} = e^{-\delta K_{Rl}} R^{\delta}_{k} + (1 - e^{-\delta K_{Rl}}) \frac{i_{Lk}}{v_{Lk}} \tag{23}$$

$$u_{d01} = u^{\delta}_{kp1} \tag{24}$$

$$u_{d02} = u_{c2}|_{t=k\delta} \tag{25}$$

The proposed scenario is composed of 3 stages:

- $t \in [0,5)s$ - The vehicle power demand is $P_l = 230.4W$ and thus $R_l = 10\Omega$;
- $t \in (5,30)s$ - $P_l = 460.8W$ and thus $R_l = 5\Omega$;
- $t \in (30,60)s$ - $P_l = 230.4W$ and thus $R_l = 10\Omega$.

Fig. 2 describes such a scenario. The load power profile is shown in Fig. 2(a), while the correspondent load conductance and the value of $1/R^{\delta}$ is depicted in Fig. 2(a).

Figs. 3 and 4 depict the system behavior for a $\delta = 0.1ms$. With this reduced sampling period, emulated and sampled-data controller performances are quite similar; in both cases

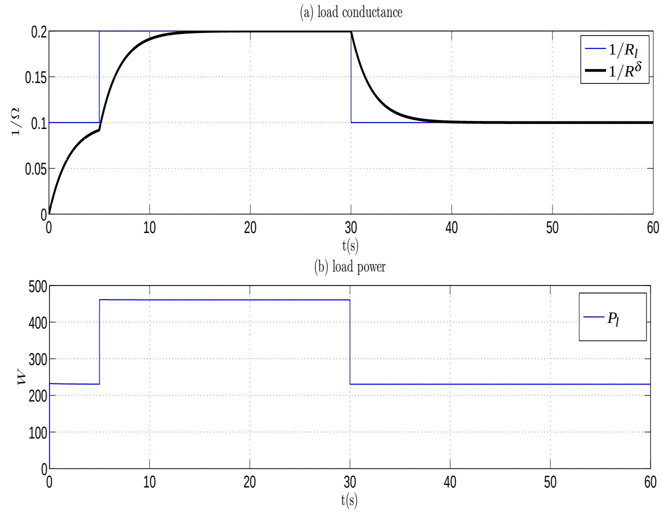

Fig. 2. Proposed scenario and $1/R^{\delta}$ evolution

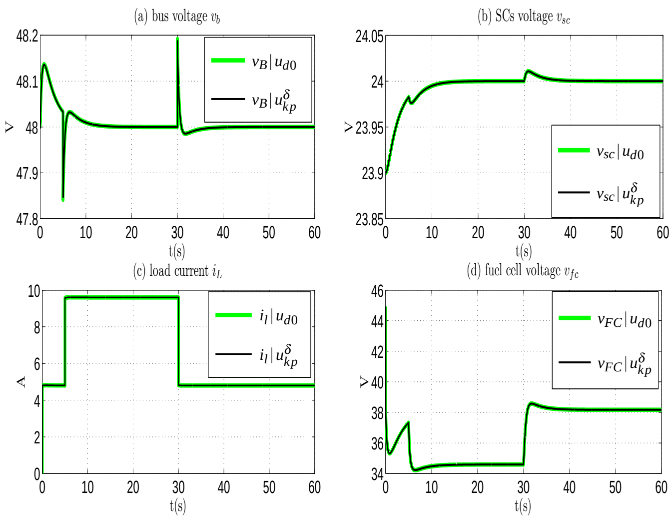

Fig. 3. System behavior for $\delta = 0.1ms$

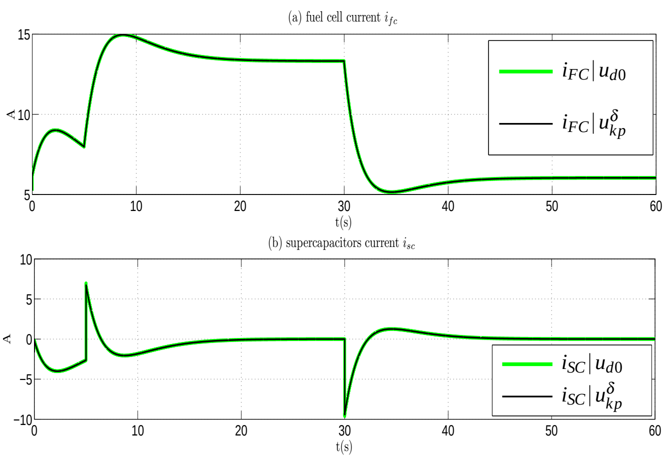

Fig. 4. i_{FC} and i_{SC} evolution for $\delta = 0.1ms$

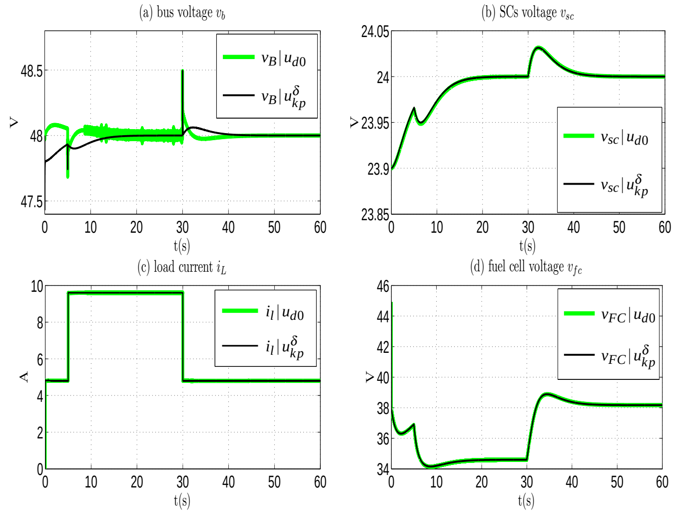

Fig. 5. System behavior for $\delta = 1ms$

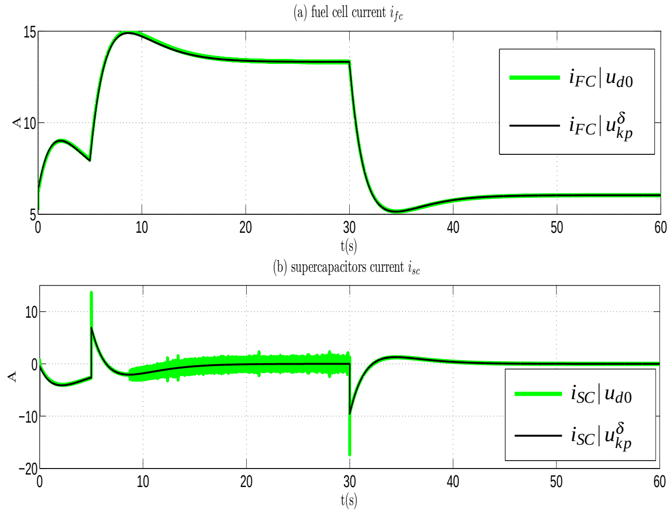

Fig. 6. i_{FC} and i_{SC} evolution for $\delta = 1ms$

when the load power demand suddenly increases, at the instant $t = 5s$, the *SCs* respond rapidly (Fig. 4(b)), ensuring the DC voltage regulation (Fig. 3(a)). Since the sensibility of i_{FC} in R_l is regulated by K_{Rl}, the fuel cell current presents a smooth variation even for fast power demands of the load, improving its state of health. When the load power demand suddenly decreases ($t = 30s$), the transient excess of current is rapidly absorbed by the SCs (Fig. 4) recharging it; at the same time i_{FC} slowly decreases.

The benefits of the proposed sampled-data strategy is observed for increasing sampling periods. Figs. 5 and 6 depict system response for $\delta = 1ms$. While sampled-data controller still ensures a good v_B regulation, with acceptable i_{FC} and i_{SC} variation, the emulated strategy degrades control objectives when an increasing load power is demanded.

In fact, when the load power demand increases ($t = 5s$), the emulated controller request a $i_{SC}^{max} = 14A$, while sampled-data controller request the half. Similarly, when the power load demand suddenly decreases ($t = 30s$), emulated strategy request a $i_{SC}^{min} = -17A$ while sampled-data controller request $i_{SC}^{min} - 9A$. Additionally, during the interval $t \in (5, 30)s$, when the load power demand is high, i_{SC} of the system under emulated controller presents strong oscillations between $2A$ and $-2A$, which reflects in an oscillation on v_B. The response of the system under sampled-data controller u^{δ}_{kp} is still good and such a strategy must be employed for applications where an increases sampling period is adopted.

VI. Conclusions

A direct sampled-data controller based on the IDA-PBC technique is developed for the energetic management of a fuel cell/supercapacitor system. Such a controller is compared with the classical implementation, so-called emulated strategy, which consists to implement the continuous-time controller through a zero order holder device. By a realistic simulation, it is verified that, instead to the emulated control, the sampled-data IDA-PBC ensures a good DC bus voltage regulation with low currents i_{FC} and i_{SC} requirements even for increasing sampling-periods.

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